- In the previous lecture we drew the FBDs for the slider-crank mechanism and established that there were nine unknowns. Now let's write out Newton's Law for each of the three rigid bodies.

Link # 2

$$+\rightarrow \sum F_x = m_2 a_{2x} = F_{12x} + F_{32x}$$

$$+\uparrow \sum F_y = m_2 a_{2y} = F_{12y} + F_{32y}$$

$$+\circlearrowleft \sum T = I_{2G}\,\alpha_2 = T_{12} + F_{12x} c_y - F_{12y} c_x - F_{32x} c_y + F_{32y} c_x$$

Link # 3

$$\sum F_x = m_3 a_{3x} = -F_{32x} + F_{43x}$$

$$\sum F_y = m_3 a_{3y} = -F_{32y} + F_{43y}$$

$$\sum T = I_{3G}\,\alpha_3 = T_{13} + F_{32x} b_y + F_{32y} b_y + F_{43x} b_y + F_{43y} b_x$$

Link # 4

$$\sum F_x = m_4 a_{4x} = -F_{43x}$$

Note: If there was friction we would add another term F_{14x} and then let $F_{14x} = \pm \mu F_{14y}$ $\rightarrow$ therefore we would still have 9 equations and 9 unknowns

$$\sum F_y = m_4 a_{4y} = F_{14y} - F_{43y}$$

$$\sum T = I_{4G}\,\alpha_4 = T_{14}$$

- Now that we have identified the relevant equations of motion we can put them into matrix form

$[A][B] = [C]$

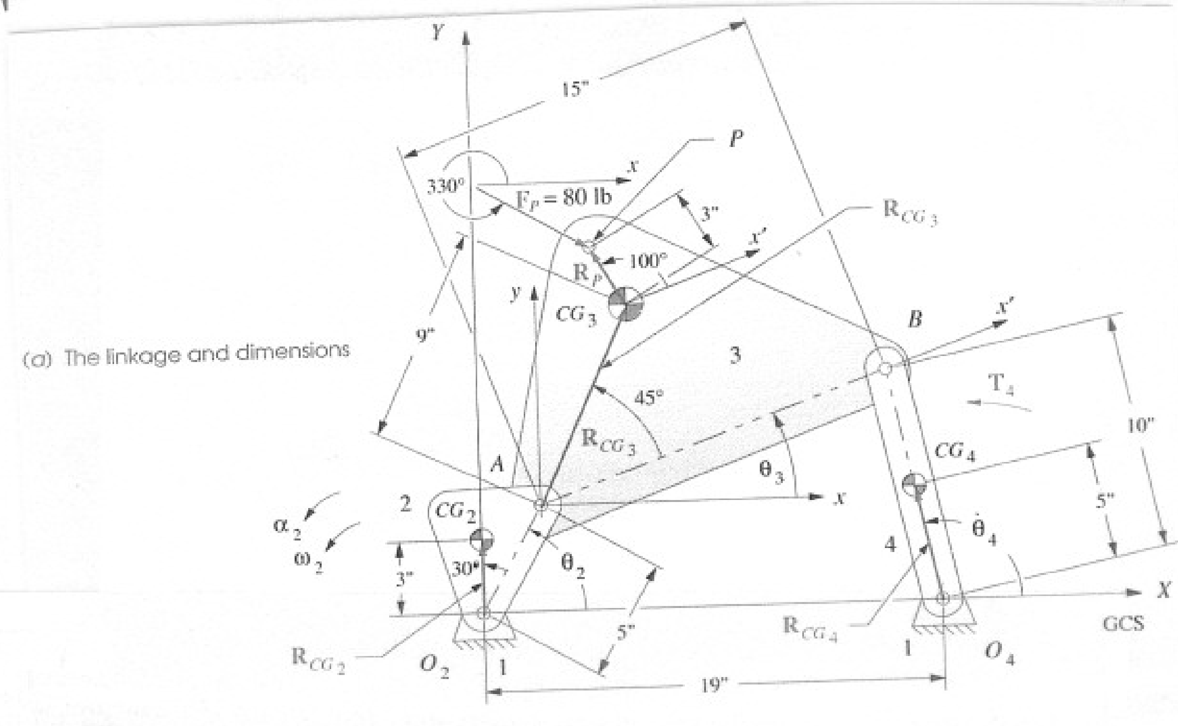
$$
\begin{bmatrix}
1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
C_y & -C_x & -C_y & C_x & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & b_y & b_x & b_y & b_x & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
F_{12x} \\ F_{12y} \\ F_{32x} \\ F_{32y} \\ F_{43x} \\ F_{43y} \\ T_{13} \\ F_{14y} \\ T_{14}
\end{bmatrix}
=
\begin{bmatrix}
m_2 a_{2x} \\ m_2 a_{2y} \\ I_{2G}\alpha_2 - T_{12} \\ m_3 a_{3x} \\ m_3 a_{3y} \\ I_{3G}\alpha_3 \\ m_4 a_{4x} \\ m_4 a_{4y} \\ I_{4G}\alpha_4
\end{bmatrix}
$$

The solution to the unknowns $= [B] = [A]^{-1}[C]$

This provides the complete dynamic force information

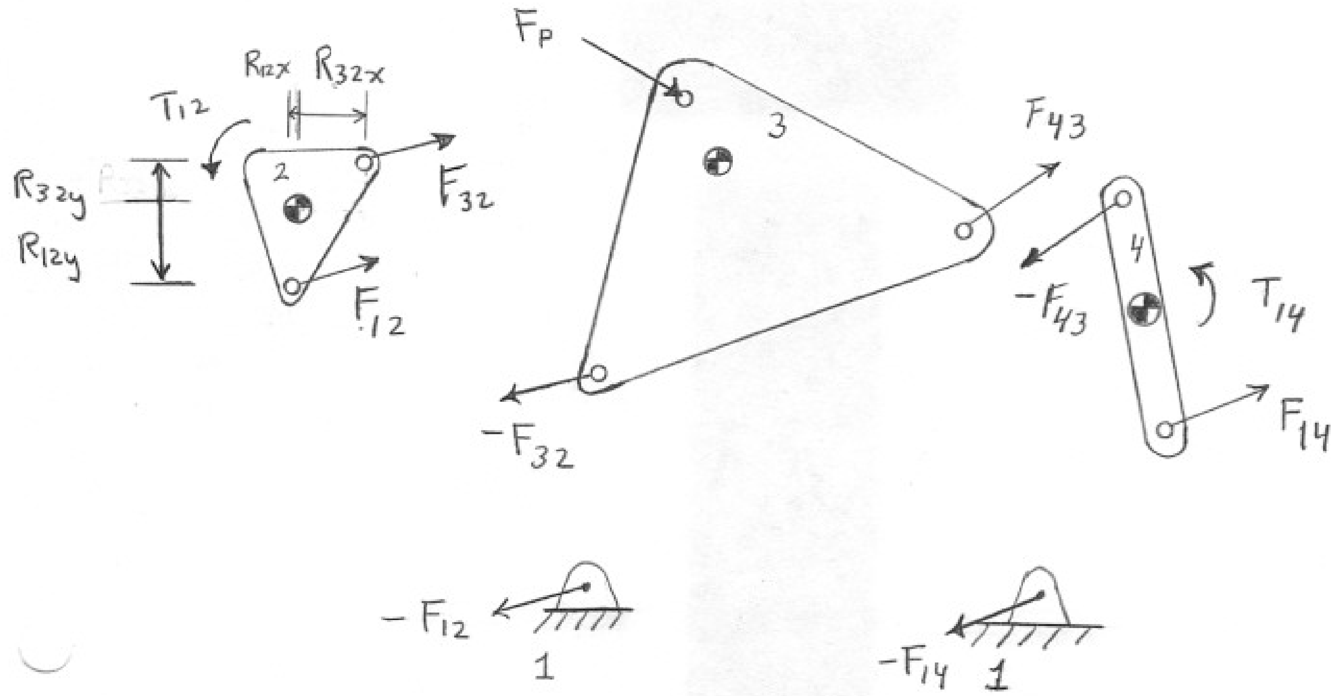
Note: since $\alpha_4 = 0$ we can see that the last equation is independent of the others and so it can be removed.

- Now let's solve the following four bar linkage problem



(a) The linkage and dimensions

The first step in solving for the forces at all the pin joints is to draw the appropriate FBDs. Of course we are assuming that we already know the kinematics (linear and angular accelerations), dimensions, masses, and moment of inertias of all rigid bodies



- Now we can write Newton's Law for each of the three rigid bodies to determine the unknowns

F_{12x}, F_{12y}, F_{32x}, F_{32y}, F_{43x}, F_{43y}, F_{14x}, F_{14y}, T_{12}

For the above case T_{14} is the input and is known.

For Link 2

$$+\rightarrow \sum F_x = m_2 a_{G2x} = F_{12x} + F_{32x}$$

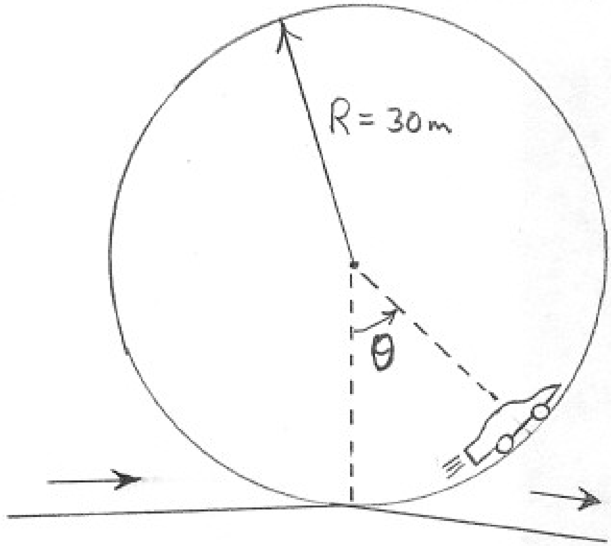
$$+\uparrow \sum F_y = m_2 a_{G2y} = F_{12y} + F_{32y}$$

$$\circlearrowleft\!+ \sum T = I_{G2}\,\alpha_2 = R_{12y}F_{12x} - R_{12x}F_{12y} - R_{32y}F_{32x} + R_{32x}F_{32y}$$

The same can be performed for Link 3 and Link 4 and the equations can be put in the form $AB = C \rightarrow B = A^{-1}C$

Example

A rocket car enters a loop with velocity $V_T = 100\ m/s$ at $30°$. The rocket generates a thrust of 10000 N. The car has a mass of 700 kg and the radius of the loop is 30m.

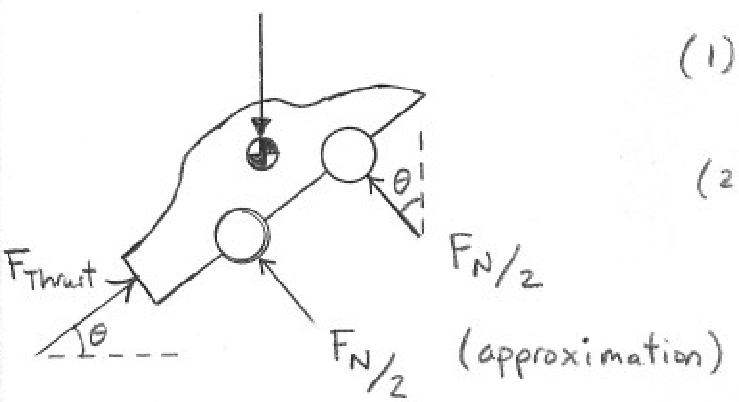


Find the normal force and the acceleration

Solution

Let's assume the outward normal is positive

The unknowns are F_N, a_T, and a_R



(1) $\Sigma F_R = m a_R = mg\cos\theta - F_N$

(2) $\Sigma F_T = m a_T = -mg\sin\theta + F_{Thrust}$

Since we know that the motion is circular, the radial acceleration is $a_R = \omega^2 R \rightarrow V_T = \omega R \rightarrow a_R = \dfrac{V_T^2}{R} = \dfrac{(100\,m/s)^2}{30\,m}$

a_R = radial acceleration = $\boxed{333\ m/s^2\ \angle\ 120° = a_R}$

Substituting into Eq. (1)

$$m\left(\frac{-v^2}{R}\right) = mg\cos\theta - F_N \rightarrow \quad F_n = mg\cos\theta + \frac{mv^2}{R}$$

$$F_n = 700\left(9.81\cos 30 + \frac{(100)^2}{30}\right) = \boxed{239\text{ kN} = F_N}$$

Substituting into Eq. (2)

$$a_T = -g\sin\theta + \frac{F_{thrust}}{m} = -9.81\sin 30 + \frac{10000}{700} = 9.38\ m/s^2$$

$$\boxed{a_T = 9.38\ m/s^2 \angle 30^\circ}$$

$$a_{TOTAL} = 333\ m/s^2 \angle 120 + 9.38\ m/s^2 \angle 30^\circ$$

We can also put the equations into the form [A][B]=[C] with the unknowns being F_N and a_T

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} F_N \\ a_T \end{bmatrix} = \begin{bmatrix} \frac{mv^2}{R} + mg\cos\theta \\ -g\sin\theta + \frac{F_{thrust}}{m} \end{bmatrix} \rightarrow \begin{array}{l} B = A^{-1}C \\ I^{-1} = I \end{array}$$

$$\begin{bmatrix} F_N \\ a_T \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} \frac{mv^2}{R} + mg\cos\theta \\ -g\sin\theta + \frac{F_{thrust}}{m} \end{bmatrix}$$